



RAN - 2103000206020033

RAN-2103000206020033**T.Y.B.Sc. (Sem.VI) Examination September - 2023****Mathematics****(MTH-603 : Real Analysis (New Course))****Time: 2 Hours]****[Total Marks: 50****सूचना : / Instructions**

(१)

नीचे दशविले निशानीवाणी विगतो उत्तरवली पर अवश्य लपववी.
Fill up strictly the details of signs on your answer book

Name of the Examination:

T.Y.B.Sc. (Sem.VI)

Name of the Subject :

Mathematics (MTH-603 : Real Analysis (New Course))

Subject Code No.: 2103000206020033

Seat No.:

Student's Signature

- (2) Figures to the right indicate marks of the question.
(3) Follow usual notations and conventions.

Q-1 Answer the following questions : (Any five).**[10]**

- (1) Prove that the sequence $\{1, 0, 1, 0, 1, 0, \dots\}$ is $(c, 1)$ summable.
(2) Suppose that $\{s_n\}_{n=1}^{\infty} = 1$ is divergent sequence of real number and $c \in R, c \neq 0$ then prove that $\{cs_n\}_{n=1}^{\infty} = 1$ diverges .
(3) Define : uniform convergence of a sequence .
(4) Define : point wise convergence of a sequence
(5) Evaluate : $\lim_{n \rightarrow \infty} \frac{1}{n} \left[\left(\frac{1}{n}\right)^2 + \left(\frac{2}{n}\right)^2 + \dots + \left(\frac{n}{n}\right)^2 \right]$.
(6) Let $(x) = x$ ($0 \leq x \leq 1$), let σ be the subdivision $\left\{0, \frac{1}{3}, \frac{2}{3}, 1\right\}$ of $[0,1]$ then compute $U[f, \sigma]$.
(7) If $f(x) = \int_0^x \sqrt{t+t^6} dt$ ($x > 0$) then find $F'(2)$.
(8) Define : lower integral for a function.

Q-2 Attempt Any Two : [10]

- (a) If s_1, s_2, s_3, \dots is $(c, 1)$ summable to S , and if $t \in R$ then prove that t, s_1, s_2, s_3, \dots is $(c, 1)$ summable to S .
- (b) Let $\{s_n\}_{n=1}^{\infty} = 1$ be a monotone sequence then prove that $\{\sigma_n\}_{n=1}^{\infty} = 1$ is a monotone sequence where $\sigma_n = \frac{S_1 + S_2 + \dots + S_n}{n}$.
- (c) Prove that a sequence that diverges to infinity cannot be $(c, 1)$ summable.

Q-3 Attempt Any Two : [10]

- (a) The sequence of function $\{f_n\}_{n=1}^{\infty}$ converges to f on E then prove that $\lim_{n \rightarrow \infty} \sup_{x \in E} |f_n(x) - f(x)| = 0$.
- (b) Let $\{f_n\}_{n=1}^{\infty}$ be sequence of real valued functions on a set E . then prove that $\{f_n\}_{n=1}^{\infty}$ is uniformly convergent on E if and only if given $\varepsilon > 0 \exists N \in I$ such that $|f_n(x) - f(x)| < \varepsilon, m, n \geq N, x \in E$.
- (c) Define point wise convergence of sequence of functions. If $g_n(x) = \frac{x}{1 + nx}$ ($0 \leq x < \infty$) then show that the sequence $\{g_n(x)\}_{n=1}^{\infty}$ converges point wise.

Q-4 Attempt Any Two : [10]

- (a) Prove that every countable subset of R^1 has measure zero.
- (b) Let f be a continuous function on $[a, b]$ then prove that $f \in R[a, b]$.
- (c) If $f \in R[a, b], g \in R[a, b]$ and if $f(x) \leq g(x)$ almost everywhere ($a \leq x \leq b$) then prove that $\int_a^b f \leq \int_a^b g$.

Q-5 Attempt Any Two: [10]

- (a) State and prove first fundamental theorem for calculus.
- (b) If f is continuous on $[a, b]$ then prove that there exists $c \in (a, b)$ such that $\int_a^b f(x) dx = f(c)(b - a)$.
- (c) Evaluate : $\lim_{n \rightarrow \infty} \frac{1}{n} \left[\sin \frac{\pi}{n} + \sin \frac{2\pi}{n} + \dots + \sin \frac{n\pi}{n} \right]$.